INTENSIFICATION OF HEAT TRANSFER AND DECREASE IN THE RESISTANCE FOR FLOW IN CHANNELS COATED WITH A MAGNETIC-LIQUID. 2. SINUSOIDAL COATING

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The basic characteristics of heat transfer for flow in a channel with a nonplanar magnetic-liquid coating are studied by means of numerical calculations.

The analytical investigations performed in Part 1 of this work demonstrated that it is in principle possible to reduce the hydraulic resistance and at the same time intensify heat transfer in a laminar flow in a channel by coating the channel walls with a low-viscosity magnetic-liquid [1]. In the case of one-dimensional flow heat transfer is intensified mainly by means of the smoothing of the velocity profile of the main flow owing to the reduction of the tangential stresses at the interface of the liquids. As a result for a given flow rate the velocity of the liquids near their interface increases, and this results in intensified refreshment of the media and intensification of heat transfer. A low-viscosity magnetic-liquid coating changes the structure of the flow in the channel in such a manner that heat transfer is intensified and at the same time the pressure losses are lower than in the case of uncoated channel. In [1] it was assumed that the form of the interface between the main flow and the coating can be assumed to be flat. This assumption is valid if sufficiently stringent restrictions are imposed on the geometry of the system (the length of the layer of magnetic liquid must be much greater than the width of the channel and the length of the starting thermal section).

In real systems with a magnetic-liquid coating the period of the coating Λ is comparable to the width of the channel. Hence in studying the hydrodynamics and heat-transfer processes in such systems the curvature of the interface between the media and the possibility of convective transfer across the channel must be taken into account. The purpose of this work is to investigate such systems and to analyze the mechanisms which are responsible for the intensification of heat transfer and are connected with the presence of a nonplanar interface between the magnetic and nonmagnetic liquids.

A two-dimensional conjugate problem of this type cannot be solved analytically. Because of this the hydrodynamics and heat transfer were investigated with the help of numerical calculations.

The geometry of the problem is shown in Fig. 1. The form of the interface between the media was assumed to be given and was approximated by sinusoid $\xi(x)=h(\delta+a\cos 2\pi x/\Lambda)$. As follows from existing experimental results [2, 3], for a given volume of magnetic liquid and configuration of the magnetic field the form of the surface of the magnetic liquid does not depend significantly on the velocity of the main flow; this is the basis for regarding the form of the interface of the media as an external parameter. This greatly simplifies the problem, since there are significant computational difficulties in solving conjugate problems with an unknown boundary and a more accurate formulation will not significantly affect the final result (the characteristics of heat transfer).

The assumption that the properties of the liquids do not depend on the temperature made it possible to separate the hydrodynamic and thermal parts of the problem. The velocity profile in the heat-transfer zone was assumed to be established; the computed velocity field was used to solve the equations of convective heat transfer.

The hydrodynamic part of the problem was solved using the stream function and the vorticity as the variables. The corresponding equations and boundary conditions have the form

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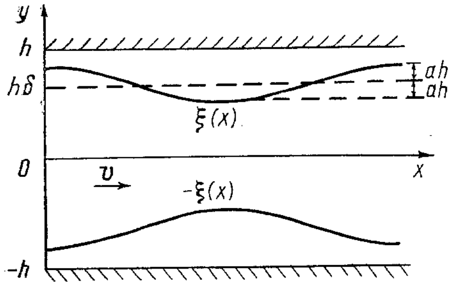

Fig. 1

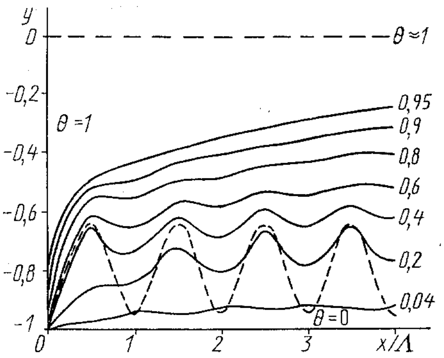

Fig. 2

Fig. 1. The geometry of the problem.

Fig. 2. The temperature field in the input section of the channel: a = 0.15; δ = 0.8; Λ = 0.5; Re_1 = 100; $\eta_1/\eta_2 = \lambda_2/\lambda_1 = 5$; Pr_1 = 100; Pr_2 = 10.

$$\Delta\psi = -\omega,\ v_x = \partial\psi/\partial y,\ v_y = -\partial\psi/\partial x;$$
$$\mathrm{Re}\,(v_x\,\partial\omega/\partial x + v_y\,\partial\omega/\partial y) = \Delta\omega;$$
$$y = 1{:}\ \psi = 1/2,\ \partial\psi/\partial y = 0;$$
$$y = \xi(x)/h{:}\ \psi = 1/2,\ \{\partial\psi/\partial y\} = 0,\ \left\{\eta\left(\omega - \frac{2\xi''}{1+\xi'^2}\,\frac{\partial\psi}{\partial y}\right)\right\} = 0;$$
$$y = -\xi(x)/h{:}\ \psi = -1/2,\ \{\partial\psi/\partial y\} = 0,\ \left\{\eta\left(\omega + \frac{2\xi''}{1+\xi'^2}\,\frac{\partial\psi}{\partial y}\right)\right\} = 0;$$
$$y = -1{:}\ \psi = -1/2,\ \partial\psi/\partial y = 0;$$
$$\psi(x+\Lambda,\ y) = \psi(x,\ y),\ \omega(x+\Lambda,\ y) = \varphi(x,\ y).$$

It can be shown that the functions $\psi(x, y)$ and $\omega(x, y)$ are odd functions of the transverse coordinate y. This property was used in the solution (it was assumed that the domain of y is $[-1, 0]$). The condition of periodicity ψ and ω is connected with the steady-state character of the flow of the liquids.

The problem (1) was solved numerically by the method of finite differences using a power-law interpolation function between the nodes [4]. The algebraic equations obtained by casting the system into a finite-difference form was solved by the Gauss–Seidel method with a relative error of 10^{-4}. The pressure losses Δp in the channel were determined from the computed fields ψ and ω.

An analogous problem was solved numerically in [2]. In so doing a transformation was made to a curvilinear coordinate system, so that the interface of the liquids would be one of the coordinate lines. The results obtained in this work are in good agreement with the results of [2] (the relative deviation is less than 1%).

The following model was used to study heat transfer accompanying flow in a channel with a sinusoidal magnetic-liquid coating: it was assumed that the temperatures of the walls are equal and constant along the channel and the temperature in the input section of the channel was assumed to be constant and different from the temperature of the walls. The corresponding equations and boundary conditions for the thermal problem have the following form:

$$\mathrm{Pe}\,(v_x\,\partial\Theta/\partial x + v_y\partial\Theta/\partial y) = \Delta\Theta;\ y = 0{:}\ \partial\Theta/\partial y = 0;$$
$$y = -1{:}\ \Theta = 0;\quad y = -\xi(x)/h{:}\ \{\lambda\partial\Theta/\partial n\} = 0,\ \{\Theta\} = 0;$$
$$x = 0{:}\ \Theta = 1;\ x \to \infty{:}\ \Theta \to 0.$$

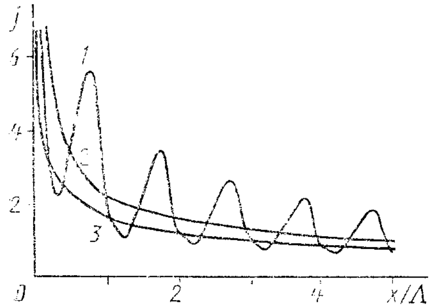

Fig. 3. The density of the heat flux through the channel wall versus the longitudinal coordinate: 1) a = 0.15; δ = 0.8; 2) 0 and 0.8; 3) 0 and 1.

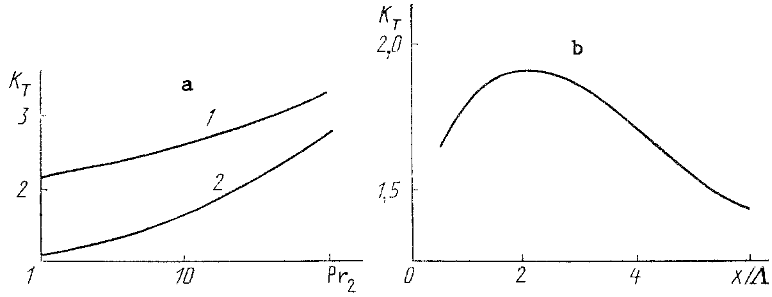

Fig. 4. The dependence of K_T on the Prandtl number of the magnetic liquid (a) (1 — a = 0.15 and δ = 0.8; 2 — a = 0 and δ = 0.8) and the period of the magnetic-liquid coating (b).

The same method as the one used in [4] to calculate the flow was employed in the numerical solution of the thermal problem.

Figure 2 shows the computed characteristic distribution of the isotherms for a flow in a channel with a periodic magnetic-liquid coating. The existence of a common origin for all isotherms is connected with the existence of a singular point x = 0, y = ± 1. Because of the nonplanar form of the interface between the media the functions y(x) describing the curves Θ = const have a nonmonotonic character.

The dependences, presented in Fig. 3, of the dimensionless density of the heat flow through the channel wall on the longitudinal coordinate indicate that the magnetic-liquid coating increases the intensity of heat transfer from the liquids. The dependence j(x) is quasiperiodic. The distance between the local extrema is approximately equal to Λ, and the average value of j(x) over a period decreases monotonically as the distance from the start of the heat-transfer zone increases. To determine the effectiveness of the magnetic-liquid coating, from the standpoint of intensification of heat transfer, the results obtained must be compared with the corresponding results for an uncoated channel.

The ratio of the Stanton numbers for a channel with and without a magnetic-liquid coating was chosen as the integral criterion for intensification of heat transfer:

$$K_T = \mathrm{St}/\mathrm{St}_0 = (Q/Q_0)/(Q'/Q_0').$$

Here Q and Q_0 are the heat fluxes through some initial section of the channel wall and through the entry section of the channel in the presence of a magnetic-liquid coating; Q' and Q_0' are the analogous quantities in the absence of a magnetic liquid. Thus K_T is the ratio of the relative amounts of heat removed from the flow with and without the magnetic liquid.

To determine the contribution of different mechanisms of heat transfer to the heat transfer process we investigated the dependence of K_T on the velocity of the main flow (Re_1), the Prandtl numbers of the liquids (Pr_1 and Pr_2), the ratio of the viscosities of the liquids η_1/η_2, as well as the form of the interface of the media (a, Λ) and the average thickness of the coating 1 — δ.

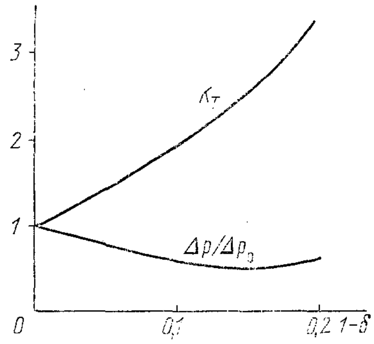

Fig. 5. K_T and the relative pressure losses $\Delta p/\Delta p_0$ versus the average thickness of the magnetic-liquid coating.

Determining the dependence of the characteristics of heat transfer on the Reynolds number makes it possible to evaluate the effect of convection on the intensity of the heat-transfer processes. By investigating the dependence of the criterion of intensification of heat transfer on the Prandtl numbers of the liquids it is possible to determine the relative contribution of the convective and conductive heat transfer in both liquids to the intensification of heat transfer. As shown in [1], the ratio of the viscosities of the main flow and the coating significantly affects both the hydrodynamics and heat transfer for flow in a channel with a flat magnetic-liquid coating. It is interesting to analyze the effect of this quantity on heat transfer in the case of a nonplanar coating. The dependence of the heat-transfer parameters on the form of the coating is related with the existence of a transverse component of the velocity of the liquids and therefore the investigation of this dependence will make it possible to take into account the convective heat transfer across the channel.

The investigation of the dependence of the characteristics of heat transfer on the Reynolds number of the main flow showed that the intensity of heat transfer from the liquids to the channel walls increases monotonically as the velocity of the main flow increases. This is connected with the intensification of convective heat transfer in the liquids.

Figure 4a shows the dependence of the criterion of investigation of the heat transfer K_T on Prandtl's number of the magnetic liquid. As Pr_2 increases the intensity of heat transfer increases, and this effect is strongest in the case of a nonplanar interface between the media; this is connected with the convective heat transfer driven by the circulating flow in the coating. In the nonmagnetic liquid heat transfer is intensified primarily as a result of convective heat transfer across the channel, since as the amplitude a increases the effect of Prandtl's number of the main flow Pr_1 on the parameters of heat transfer increases substantially. In particular, for a large amplitude an increasing Pr_1 results in a significantly bigger increase in the criterion of intensification of heat transfer K_T.

The intensity of heat transfer also increases as a result of the increase in the flow velocity of the liquids near the interface and the smoothing of the velocity profile in the main flow as the viscosity of the magnetic-liquid coating decreases.

It is intersting to investigate the effect of the perid of the coating Λ on the intensity of heat transfer. Figure 4b shows the characteristic dependence $K_T(\Lambda)$, whose main feature is the existence of an optimal period of the coating (approximately equal to the width of the channel), for which the intensity of heat transfer is maximum. The existence of this maximum is connected with the competition between two factors: on the one hand, for intensification of heat transfer the transverse component of the velocity of the liquids must increase, as happens when the period decreases; on the other hand, an increase in the velocity of the liquids at the interface between them, which an increase in the period of the coating can bring about, intensifies the heat transfer.

The most important advantage of the method of intensification of heat transfer with the help of magnetic-liquid coatings over the traditional methods is illustrated in Fig. 5. Figure 5 shows the criterion of the intensification of heat transfer K_T and the relative pressure losses $\Delta p/\Delta p_0$ as a function of the average thickness of the coating. As follows from the results obtained, as the thickness of the coating increases the intensity of heat transfer increases monotonically, while the pressure losses at first decrease and then, after reaching a minimum, increase; this is connected with the decrease in the effective cross section of the channel and therefore, for a fixed flow rate, with an increase in viscous dissipation in the liquids. Thus there exists a range of coating thicknesses in which significant intensification of heat transfer with a simultaneous decrease of the hydraulic resistance of the channel are possible.

Analysis of the numerical results established the criterional relations between the criterion of intensification of heat transfer and the values of the parameters of the system. In particular, for $a = 0.15$, $\delta = 0.8$, and $\lambda_2/\lambda_1 = 5$ the dependence of K_T on the rest of the parameters can be described by the following expression with a relative error of up to 10%:

$$K_T = 1 + 3\cdot 10^{-3}\,\frac{\Lambda L^{0.41}}{\Lambda^2+\Lambda+4}\,\mathrm{Re}_1^{0.37}\left[\mathrm{Pe}_1^{0.35} + 6.5\left(\frac{\eta_1}{\eta_2}\right)^{0.43}\mathrm{Pe}_2^{0.37}\right].$$

The ranges of the parameters are as follows: $1 \le L/\Lambda \le 8$, $0 \le \eta_1/\eta_2 \le 20$, $1 \le \mathrm{Re}_1$, Pr_1, $\mathrm{Pr}_2 \le 100$.

Quantitative estimates show that if VM-1 vacuum oil is used as a heat-transfer agent and a water-based magnetic liquid with a saturation magnetization of the order of 20 kA/m is used for the coating, then in a 1 cm wide channel with the heat-transfer agent flowing with a velocity of the order of 1 cm/sec heat transfer can be intensified by a factor of 2 to 4 and at the same time the hydraulic resistance can be halved.

Thus our analysis indicates that magnetic-liquid coatings are effective for simultaneously reducing the hydraulic resistance and intensifying heat transfer in laminar flow of a nonmagnetic heat-transfer agent in a channel. The basic mechanisms of intensification of heat transfer are as follows: convection across the channel in the main flow, circulation flow in the magnetic-liquid coating, smoothing of the velocity profile in the main flow, and increase of the flow velocity of the liquids near the interface of the liquids; this is connected with the decrease in the tangential stresses at the boundary of the main flow when the magnetic liquid has a low viscosity.

NOTATION

x and y, dimensionless coordinates along and across the flow, respectively; v, dimensionless velocity of the liquid; h, width of the channelm in m; η, coefficient of dynamic viscosity, Pa·sec; λ, thermal conductivity, W/(m·K); Θ, dimensionless temperature; $1 - \delta$, a, and Λ, dimensionless average thickness, amplitude, and period of the magnetic-liquid coating; $\{A\}$, difference of the values which A assumes on both sides of the interface of the liquids. The indices 1 and 2 refer to the main flow and the magnetic-liquid coating, respectively.

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